

B.Sc. Semester - 1 (CBCS) Examination
Jan/Feb.-2022 (OLD COURSE)
MATHEMATICS (CORE)

Time: 1:30 Hours

Marks:42

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (A) Answer any one of the following questions.**[07]**

- (1) State and prove Roll's theorem and explain it geometrically.
- (2) Expand e^x in ascending powers of $(x - 3)$.

Q.1 (B) Answer any one of the following questions.**[04]**

- (1) For $f(x) = \log x$, $g(x) = \tan^{-1} x$, $(x \in [1, 2])$ show that $\frac{3}{2} < \frac{\log 2}{\cot^{-1} 3} < 3$.
- (2) Show that the function $f(x) = 2 - 3x + 6x^2 - 4x^3$ is strictly decreasing in every interval.

Q.1 (C) Answer any three of the following questions.**[03]**

- (1) LUB $[0, 2] =$ _____.
- (2) True / False : Lagrange's theorem is a particular case of Roll's theorem.
- (3) Write coefficient of x^4 in the Maclaurin's series expansion of $\sin x$.
- (4) True / False: Roll's theorem can be applied to the function $f(x) = |x|$ in $[-1, 1]$.
- (5) State Cauchy's theorem.

Q. 2 (A) Answer any one of the following questions.**[07]**

- (1) Solve : $e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$.
- (2) Solve : $(x + 2y^3) \frac{dy}{dx} = y$.

Q.2 (B) Answer any one of the following questions.**[04]**

- (1) If $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3} = b$, then find values of a and b .
- (2) Evaluate $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$.

Q.2 (C) Answer any three of the following questions.**[03]**

- (1) Form differential equation from the equation $y = mx$ where m is a constant.
- (2) Find I.F. of the differential equation $\frac{dy}{dx} + \frac{2y}{x} = \log x$.
- (3) Evaluate : $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$.
- (4) $\lim_{x \rightarrow 0} x^2 \log(x) =$ _____.
- (5) $\lim_{x \rightarrow 0} \frac{\tan^{-1} 2x}{x} =$ _____.

Q.3 (A) Answer any one of the following questions.**[07]**

- (1) State and prove necessary and sufficient condition for the differential equation $M(x, y) \, dx + N(x, y) \, dy = 0$ to be exact.
- (2) Solve : $(2xy + y - \tan y) \, dx + (x^2 - x \tan^2 y + \sec^2 y) \, dy = 0$

Q.3 (B) Answer any one of the following questions.**[04]**

- (1) Solve : $y^2(y - xp) = x^4 p^2$.
- (2) Solve : $\tan(y - 2px) = xp^2$.

Q.3 (C) Answer any three of the following questions.**[03]**

- (1) True / False : Linear differential equation is a particular case of Bernoulli's differential equation.
- (2) Write solution of the differential equation $y = px + \sin p$.
- (3) Solve : $p^2 - p - 6 = 0$.
- (4) Write an example of an exact differential equation.
- (5) Write general form of Clairaut's differential equation.

Q.4 (A) Answer any one of the following questions.**[07]**

(1) Solve $(D^2 - 4D + 3)y = \cos 2x \sin 3x$.

(2) In usual notations prove that $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$, Where V is a function of x .

Q.4 (B) Answer any one of the following questions.**[04]**

(1) Find particular integral of the equation $(D^2 - 4D + 4)y = e^{-4x} + 5 \cos 3x$.

(2) Solve $(D - 1)^2 y = x e^x \sin x$.

Q.4 (C) Answer any three of the following questions.**[03]**

(1) $\frac{1}{D^2-3} \sin 2x = \underline{\hspace{2cm}}$.

(2) $\frac{1}{D-1} e^x x = \underline{\hspace{2cm}}$.

(3) Find C.F. of the differential equation $(D^2 - 4)(D - 2)y = \sin x$.

(4) Define: Auxiliary equation

(5) $\frac{1}{D} e^{4x} = \underline{\hspace{2cm}}$.

Q.5 (A) Answer any one of the following questions.**[07]**

(1) Solve : $(x^4 D^3 + 2x^3 D^2 - x^2 D + x) y = 1$.

(2) Solve : $(x^2 D^2 - 2)y = x^2 + \frac{1}{x}$.

Q.5 (B) Answer any one of the following questions.**[04]**

(1) Find P.I. of $(x^2 D^3 + x D^2)y = 1$.

(2) Find C.F. of $(x^2 D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}$.

Q.5 (C) Answer any three of the following questions.**[03]**

(1) Write general form of homogeneous linear differential equation.

(2) $\frac{1}{\theta^2-3\theta+2} x^3 = \underline{\hspace{2cm}}$.

(3) C.F. of $(x^2 D^2 + 7xD + 5)y = e^x$ is $\underline{\hspace{2cm}}$.

(4) Find auxiliary equation of $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = x^3$.

(5) Find C. F. of $\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{\log x}{x^2}$.
